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3

## All Targets Near and Far:

 Accuracy, Precision and Significant Figures
## Accuracy and Precision

All measurements are estimates of some true value. The validity of any measurement is primarily due to the nature of the device used to make the measurement and partly due to the manner in which data is collected.

Model 1
Accuracy and Precision


A measured value that matches the true value is a "Bull's-eye".

Name $\qquad$

## Critical Thinking Questions

## CTQ 1

Using Model 1, define accuracy using a grammatically correct sentence.

## CTQ 2

Define precision using a grammatically correct sentence.

## CTQ 3

High precision implies high accuracy, true or false?

## CTQ 4

High accuracy implies high precision, true or false?

A ccording to the U.S. Mint, the mass of a U.S. quarter ( $\$ 0.25$ ) is 5.670 g . Four students measured the same coin (Table 1).

| Table 1. Mass of U.S. Quarter |  |  |  |
| :---: | :---: | :---: | :---: |
| I | II | III | IV |
| 5.010 g | 5.670 g | 5.673 g | 5.669 g |
| 5.010 g | 5.589 g | 5.674 g | 5.672 g |
| 5.011 g | 5.801 g | 5.673 g | 5.672 g |
| 5.011 g | 5.800 g | 5.669 g | 5.671 g |
| 5.010 g | 5.712 g | 5.669 g | 5.669 g |

## CTQ 5

Which data set is the most precise? I II III IV

## CTQ 6

What information is required to determine the accuracy of any data set?

## CTQ 7

Explain how you could determine (mathematically) which set is the most accurate?

## CTQ 8

Which data set is the most accurate? I II III IV

Name $\qquad$

## B. Uncertainty (or Error)

Every measurement is an estimate of the actual value because every measurement contains a degree of uncertainty or error. In this context, error does not mean "mistake". Error (uncertainty) is a mathematically determined variance between individual measurements that arises when repeated measurements are made on the same sample or object. For example, four students weighed a U.S. dime (\$0.10) multiple times using different types of balances (Table 2).

| Table 2. Mass of U.S. Dime |  |  |  |
| :---: | :---: | :---: | :---: |
| I | II | III | IV |
| 2.2 g | 2.29 g | 2.268 g | 2.2681 g |
| 2.1 g | 2.26 g | 2.269 g | 2.2689 g |
| 2.2 g | 2.21 g | 2.260 g | 2.2683 g |
| 2.3 g | 2.23 g | 2.267 g | 2.2682 g |
| 2.3 g | 2.27 g | 2.268 g | 2.2680 g |

The measurements in Set I contain two digits. The first digit (the one’s place) is called the reproducible digit and the second digit in the tenth's place (0.1) is called the doubtful digit.

## CTQ 9

Formulate a definition for the term reproducible digit.

CTQ 10
Formulate a definition for the term doubtful digit.

## CTQ 11

Identify the doubtful digit in each data set in Table 2.
The doubtful digit is:

| data <br> set | the one's <br> place | one-tenth's <br> place | hundredth's <br> place | thousandth's <br> place | ten-thousandth’s <br> place |
| :---: | :---: | :---: | :---: | :---: | :---: |
| I |  |  |  |  |  |
| II |  |  |  |  |  |
| III |  |  |  |  |  |
| IV |  |  |  |  |  |

Name $\qquad$

## CTQ 12

What is the uncertainty in each data set from Table 2?

| Data Set | $+/-0.1 \mathrm{~g}$ | $+/-0.01 \mathrm{~g}$ | $+/-0.001 \mathrm{~g}$ | $+/-0.0001 \mathrm{~g}$ |
| :---: | :--- | :--- | :--- | :--- |
| I |  |  |  |  |
| II |  |  |  |  |
| III |  |  |  |  |
| IV |  |  |  |  |

## CTQ 13

An increase in precision is:
a. an increase in uncertainty
b. a decrease in uncertainty

## C. Significant Figures (significant digits, sig figs), Exact Numbers and Those Pesky Zeros

Cignificant figures are those digits in any number that designate the level of Sprecision.
Sig figs include all reproducible digits plus the first doubtful digit.

| Table 3. Mass of U.S. Nickel |  |  |  |
| :---: | :---: | :---: | :---: |
| I | II | III | IV |
| 5.0 g | 5.01 g | 5.001 g | 5.0001 g |
| 5.0 g | 5.00 g | 5.002 g | 5.0009 g |
| 5.0 g | 5.01 g | 5.002 g | 5.0002 g |
| 5.1 g | 5.01 g | 5.000 g | 5.0006 g |
| 5.0 g | 5.02 g | 5.002 g | 5.0003 g |

## CTQ 14

According to the definition above, indicate how many sig figs are in each measurement:
a. Set I $\qquad$
b. Set II $\qquad$
c. Set III $\qquad$
d. Set IV $\qquad$
$\qquad$

Exact numbers contain no error (uncertainty) since these numbers are not measurements; in other words, an exact count is not an estimate. For example, the number of people in a family of four is an exact number (Table 4).

| Table 4. Measurements versus Exact Numbers |  |  |  |
| :--- | :--- | :--- | :--- |
| Measurements (estimates) |  | Exact Numbers |  |
| 0.00024 kg | U.S. population | 24 students in <br> classroom | 5 fingers <br> each hand |
| 2 cups of milk | volume of a 1-L <br> water bottle | 17 one-liter <br> water bottles | 1 dozen $=12$ |
| 18 mph | $9.3 \times 10^{6}$ miles | $2.54 \mathrm{~cm}=1$ inch | $100 \mathrm{~cm}=1$ meter |

Non-zero digits are always significant but zeros may or may not be significant. In fact, zeros cause the most confusion when dealing with significant digits. Examine Table 5.

| Table 5. Zeros are not always significant |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| $\mathbf{~ s i g ~ f i g ~}$ | $\mathbf{2}$ sig fig | $\mathbf{3}$ sig fig | $\mathbf{4} \mathbf{~ s i g}$ fig | $\mathbf{5}$ sig fig |
| 3 kg | 2.9 kg | 2.98 kg | 2.982 kg | 2.9824 kg |
| 80 km | 81 km | 80.7 km | 80.76 km | 80.761 km |
| 0.06 g | 0.055 g | 0.0550 g | 0.05503 g | 0.055039 g |
| 200 L | 2.0 | 20.0 | 200.0 L | 200.00 L |
| $3 \times 10^{8} \mathrm{~m}$ | $3.0 \times 10^{8} \mathrm{~m}$ | $3.00 \times 10^{8} \mathrm{~m}$ | $3.008 \times 10^{8} \mathrm{~m}$ | $3.0081 \times 10^{8}$ <br> m |

## CTQ 15

Using Table 5 and the examples shown here, answer true or false:

|  | examples | T | F |
| :--- | :--- | :--- | :--- |
| Zeros between non-zero digits | 80.7 |  |  |
| are significant | $3.008 \times 10^{8}$ |  |  |
| Zeros at the beginning of a number | 0.06 |  |  |
| are significant: | 0.055 |  |  |
| Zeros in the coefficient for scientific notation | $3.0 \times 10^{8}$ |  |  |
| are significant | $1.00 \times 10^{-3}$ |  |  |
| Trailing zeros in numbers written with a decimal point | 200.0 |  |  |
| are significant | 8.750 |  |  |
| Trailing zeros in numbers written without a decimal point | 80 |  |  |
| are significant | 200 |  |  |

Name $\qquad$

## D. Calculations Involving Sig Figs and Rounding Numbers

Acalculation cannot change the level of uncertainty. There are two rules to observe, one for multiplication and one for addition:

## Rule 1

When multiplying (or dividing) measurements, the answer must contain the same number of sig figs as the starting quantity with the fewest sig figs.

For example, calculate the mileage for an automobile based on the following data, 108.4 miles driven using 3.5 gallons of fuel:
$\frac{108.4 \mathrm{mi}}{3.5 \mathrm{gal}}=30.97142857 \mathrm{mpg}=31 \mathrm{mpg}$
Since the denominator has fewer sig figs than the numerator, the answer must be rounded to two sig figs.

## Rule 2

When adding (or subtracting) measurements, the answer must contain the same number of decimal places as the starting quantity with the fewest decimal places.

Consider the addition of volume in this example:
$7.6 \mathrm{~mL}+125 \mathrm{~mL}=132.6 \mathrm{~mL}=133 \mathrm{~mL}$ count decimal places, not sig figs
In this case, one measurement ( 125 mL ) does not have any digits past the one’s place and the answer must be rounded to the nearest one milliliter.

When calculations generate trailing zeros, conversion to scientific notation clarifies the number of sig figs allowed in the answer:
$730 \mathrm{~cm}+880 \mathrm{~cm}=1610 \mathrm{~cm}=1.610 \times 10^{3} \mathrm{~cm}$
Trailing zero is allowed since precision is to the nearest one's place.
$525 \mathrm{~g} \mathrm{x} 4=2100 \mathrm{~g}=2 \times 10^{3} \mathrm{~g}$
Trailing zeros are not allowed because the precision is limited to one sig fig.
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Rounding Numbers: Calculations often generate non-significant digits, especially calculations done with an electronic calculator; these values must be rounded to the correct number of sig figs. Once it is determined how many sig figs are allowed, all remaining digits to the right must be dropped. However, if the first digit to be dropped is 5 or greater, then the preceding digit is rounded up by one, then the extra digits are dropped.

In these examples, two sig figs are allowed in the answer:
$2.1 \times 3.5=7.35=7.4$ round up the doubtful digit by one then truncate extra digit
$2.1 \times 3.4=7.14=7.1$ do not round up, truncate extra digit
In this example, three sig figs are allowed in the answer:
$252 / 1.401=179.8715203=180=1.80 \times 10^{2}$ round up and convert to sci.
notation
Notice the zero generated after rounding- this is the most ambiguous type of zero, a trailing zero with no decimal point. But inspection of the calculation reveals this to be a sig fig and should be included in the final answer. Conversion to scientific notation removes all doubt that this zero is significant and should be included in the final answer.

More about zeros. Frequently, rounding off numbers produces zeros which may or may not be significant. Scientific notation is the best method to remove all ambiguity and to eliminate placeholder zeros. Below are four more examples of how to deal with zeros.

1. Zeros generated to the left of the decimal point after rounding off become placeholder zeros (to hold the decimal point) but are not significant digits.


Name $\qquad$
2. Zeros generated to the right of the decimal point after rounding off may be significant depending on how many digits are allowed in the final answer.
rounded to 3
sig figs

3. Beware of Zeros that "disappear" from electronic calculators. Compare the addition problem below done by hand versus using a calculator.

Addition by pencil:
$87.72+2.28=90.00$ (keep 2 decimal places)

Addition by calculator:
$87.72+2.28=90$
4. Always include a "courtesy zero" for numbers that begin with a decimal point. Your instructor will love you for this!

Courtesy zeros are not significant digits.
Examples: (0.01 not .01) (0.005 not .005) (0.100 not .100 and not .1)

## Exercises With Sig Figs and Rounding

1. Assume each number is a measured value. How many sig figs are in each number?
$\qquad$ a. 93 million $\qquad$ g. 300 $\qquad$ m. $1.81 \times 10^{-5}$
$\qquad$ b. $93,000,000$ $\qquad$ h. 300.0
$\qquad$ n. $6.0 \times 10^{0}$
$\qquad$ c. $9.30 \times 10^{7}$ $\qquad$ i. 0.607
$\qquad$ o. 0.00023
$\qquad$ d. $93,125,479$
$\qquad$ j. 1.277
$\qquad$ p. 23,004
$\qquad$ e. 0.677
$\qquad$ k. 6089
$\qquad$ q. 873.20
$\qquad$ f. 0.0677
$\qquad$ l. $6.700 \times 10^{-2}$
$\qquad$ r. 0.00500

Name $\qquad$
2. Round each number to 3 sig figs; write answers as decimal numbers and in scientific notation.

$$
\text { decimal value } \quad \text { scientific notation }
$$

a. 30,010
b. 30.54
c. 609.7
d. 5.2377
e. 0.1278
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
.
3. Solve each calculation and round the answer to the correct number of sig figs. Use good judgment and convert to scientific notation where appropriate- to preserve zeros that are significant or to eliminate placeholder zeros. Include units where indicated.
a. $0.433 \times 7.6889$
b. $7.22 \times 19.4$
c. 989.17 / 23.1
d. $9.94+0.05$
e. $6.94-0.072$
f. $134.9-0.05$
g. $(2.21+0.09) \times 12.75$
h. $(2.43+0.07) \times 34.5$
i. $(2.5 \mathrm{~cm} \times 0.65 \mathrm{~cm})$
j. Convert 65.4 g into kg

All Targets Near and Far
POGIL ACTIVITY. 3
Name $\qquad$
NOTES

